Predicate Logic: Exercise 1

Consider the following predicate:

$$\forall\, x, y \bullet x \in \mathbb{N} \land y \in \mathbb{N} \Rightarrow x * y > 0$$

Choose **all** statements that are **correct**.

1. It is a theorem, provable by (5, 4).
2. It is a theorem, provable by (2, 3).
3. It is not a theorem, witnessed by (5, 0).
4. It is not a theorem, witnessed by (12, -2).
5. It is not a theorem, witnessed by (12, 13).

Predicate Logic: Exercise 2

Consider the following predicate:

$$\exists\, x, y \bullet x \in \mathbb{N} \land y \in \mathbb{N} \land x * y > 0$$

Choose **all** statements that are **correct**.

1. It is a theorem, provable by (5, 4).
2. It is a theorem, provable by (2, 3).
3. It is a theorem, provable by (-2, -3).
4. It is not a theorem, witnessed by (5, 0).
5. It is not a theorem, witnessed by (12, -2).
6. It is not a theorem, witnessed by (12, 13).

Nested Logical Quantifiers

$$\forall\, i \bullet i \in \mathbb{Z} \Rightarrow (\exists\, j \bullet j \in \mathbb{N} \land i + j = 0\ )$$

$$\forall\, i \bullet i \in \mathbb{N} \Rightarrow (\exists\, j \bullet j \in \mathbb{Z} \land i + j = 0\ )$$

$$\exists\, i \bullet i \in \mathbb{N} \Rightarrow (\forall\, j \bullet j \in \mathbb{Z} \land i \cdot j > 0)$$

$$\exists\, i \bullet i \in \mathbb{N} \Rightarrow (\forall\, j \bullet j \in \mathbb{Z} \land i \cdot j \geq 0\ )$$

Use of Model Checking in Industry

Pentium FDIV bug: https://en.wikipedia.org/wiki/Pentium_FDIV_bug

*The Pentium FDIV bug is a hardware bug affecting the **floating-point unit (FPU)** of the early Intel Pentium processors. Because of the bug, the processor would return incorrect binary floating point results when dividing certain pairs of high-precision numbers.*

*In December 1994, Intel **recalled** the defective processors ... In its 1994 annual report, Intel said it incurred "**a $475 million pre-tax charge** ... to recover replacement and write-off of these microprocessors."*

*In the aftermath of the **bug** and subsequent **recall**, there was a marked increase in the use of formal verification of hardware floating point operations across the **semiconductor industry**. Prompted by the discovery of the bug, a technique ... called "word-level **model checking**" was developed in 1996. Intel went on to use **formal verification** extensively in the development of later CPU architectures. In the development of the Pentium 4, symbolic trajectory evaluation and **theorem proving** were used to **find a number of bugs that could have led to a similar recall incident** had they gone undetected.*

Formal Verification: Proof Based vs. Check Based